Wiederholung: Faltungssumme

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k] \cdot \delta[n-k] \tag{1}$$

$$y[n] = T\left\{ \sum_{k=-\infty}^{+\infty} x[k] \cdot \delta[n-k] \right\} \tag{2}$$

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k] \cdot T\left\{\delta[n-k]\right\} \tag{3}$$

$$h[n] = T\left\{\delta[n]\right\} \qquad h[n-k] = T\left\{\delta[n-k]\right\} \tag{4}$$

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k] \cdot h[n-k] = (x * h)[n] \tag{5}$$

Eigenschaft Faltungssumme: Kommutativität

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k] \cdot h[n-k] = (x * h)[n] \tag{6}$$

$$y[n] = \sum_{k=-\infty}^{+\infty} h[k] \cdot x[n-k] = (h * x)[n] \tag{7}$$

$$(x * h)[n] = \sum_{k=-\infty}^{+\infty} x[k] \cdot h[n-k] \tag{8}$$

$$m = n - k \tag{9}$$

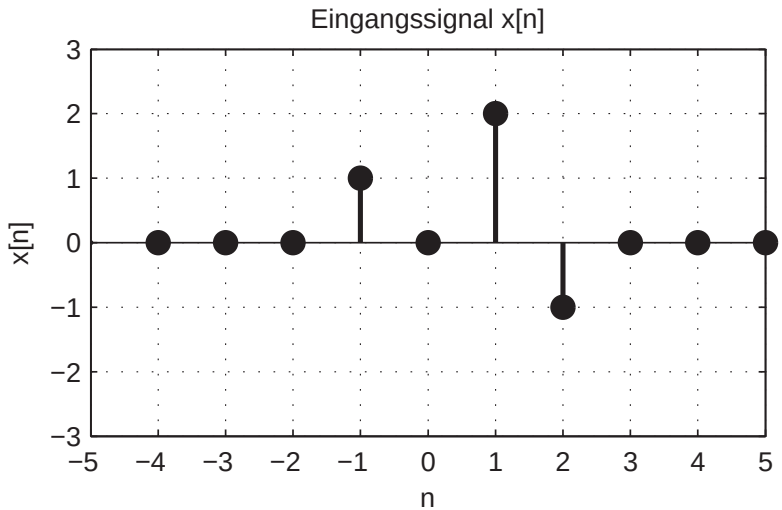
$$(h * x)[m] = \sum_{m=-\infty}^{+\infty} h[m] \cdot x[n-m] \tag{10}$$

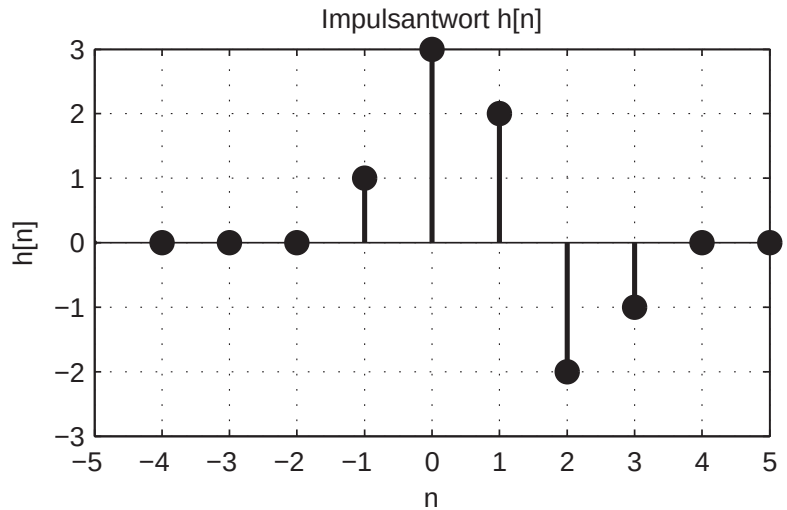
$$(x * h)[n] = (h * x)[n] \tag{11}$$

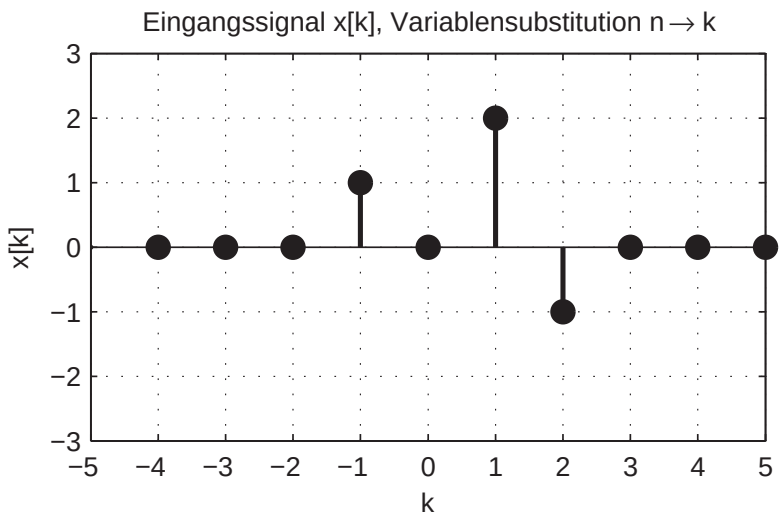
Grafische Veranschaulichung mit

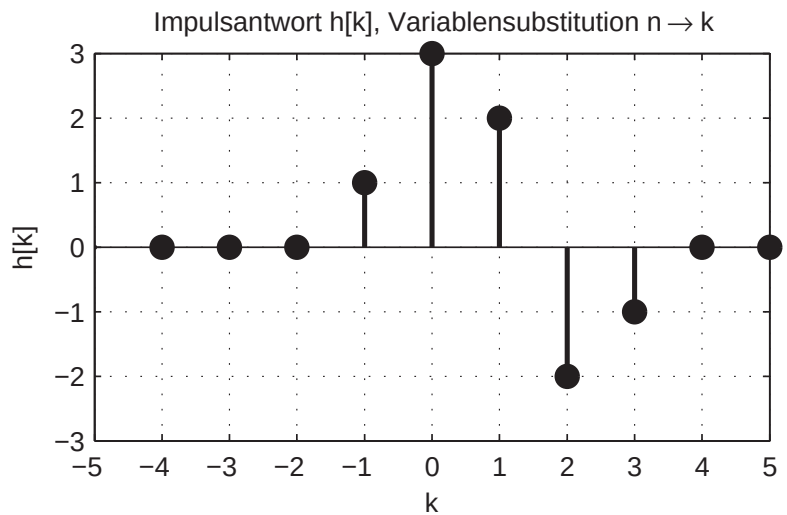
$$y[n] = \sum_{k=-\infty}^{+\infty} h[k] \cdot x[n-k] = (h * x)[n] \tag{12}$$

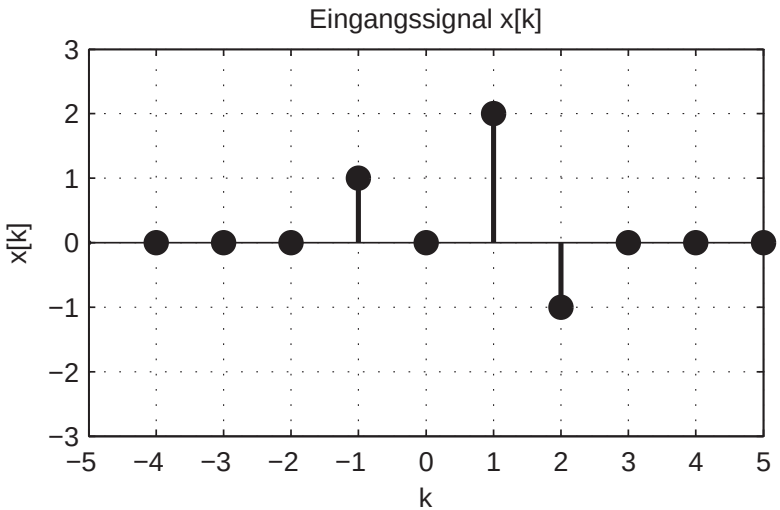
`http://www.nt.e-technik.uni-erlangen.de/~rabe/SYSTOOL/SYSTOOL2.03/conv.htm`

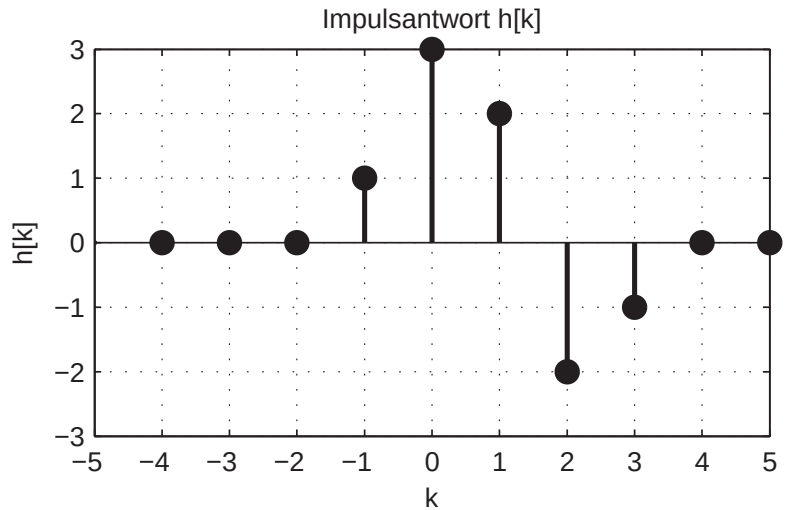

Eingangssignal x[n]
x[n]
n

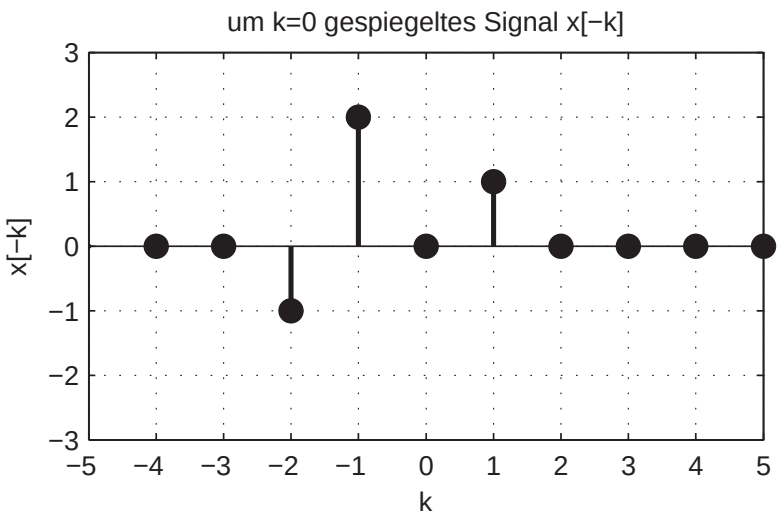

Impulsantwort h[n]
h[n]
n

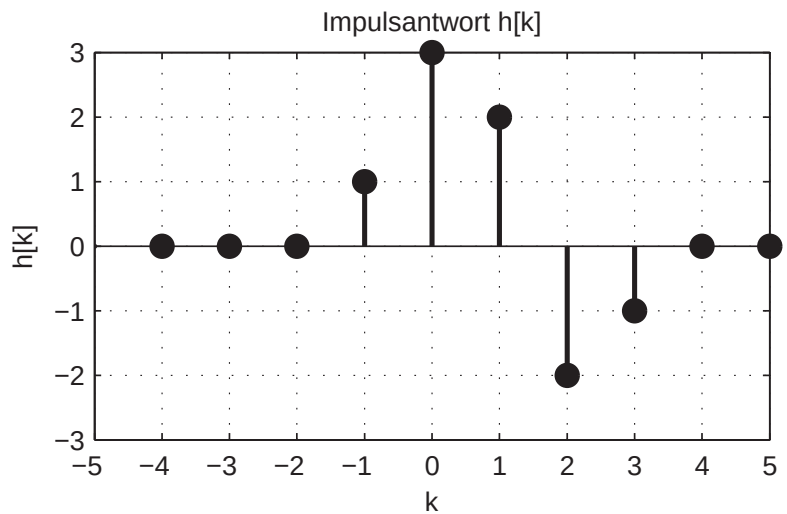

Eingangssignal x[k], Variablensubstitution n → k
x[k]
k

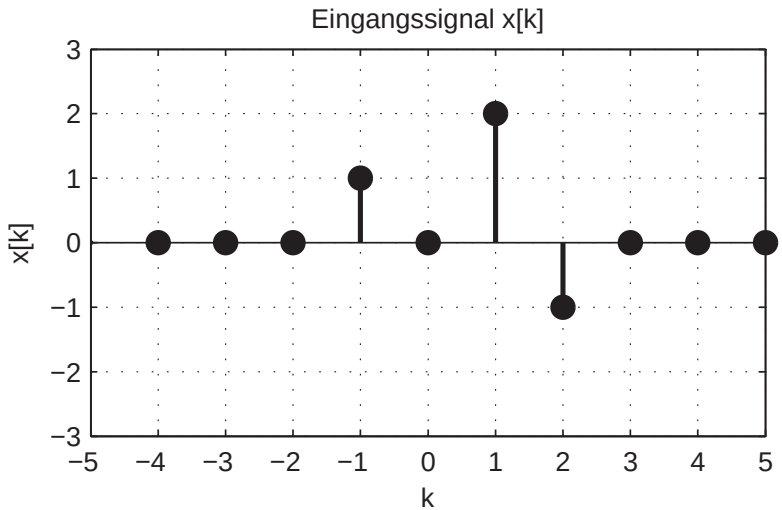

Impulsantwort h[k], Variablensubstitution n → k
h[k]
k

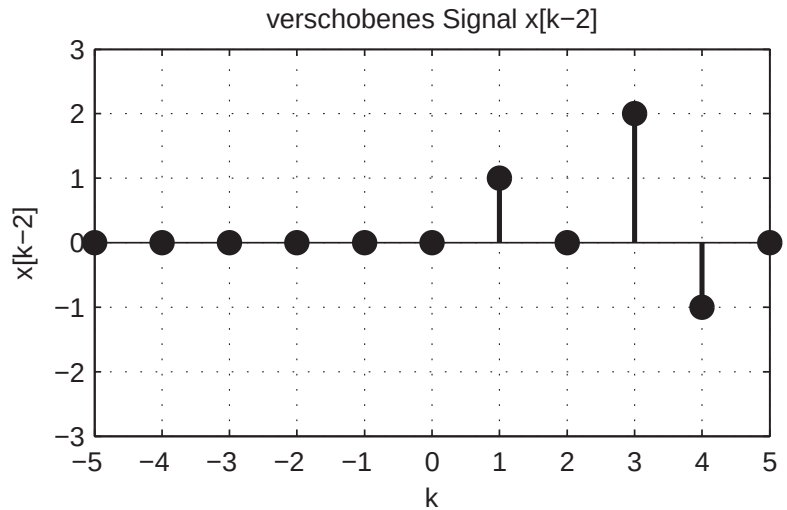

Eingangssignal x[k]
x[k]
k

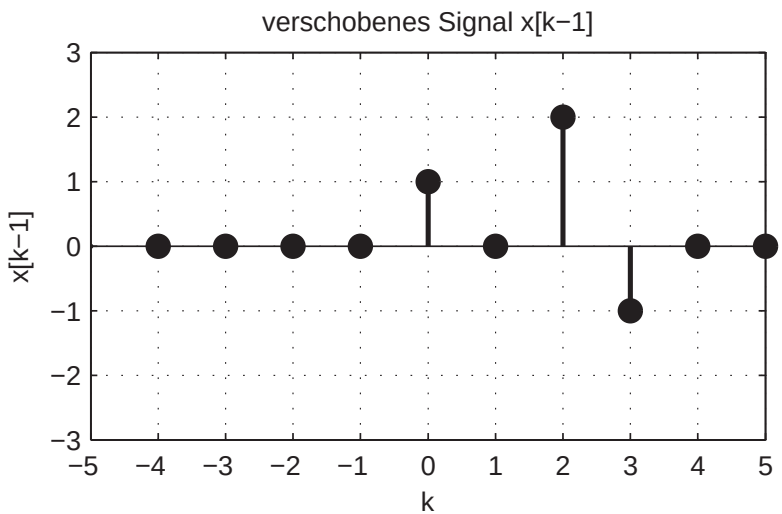

Impulsantwort h[k]
h[k]
k

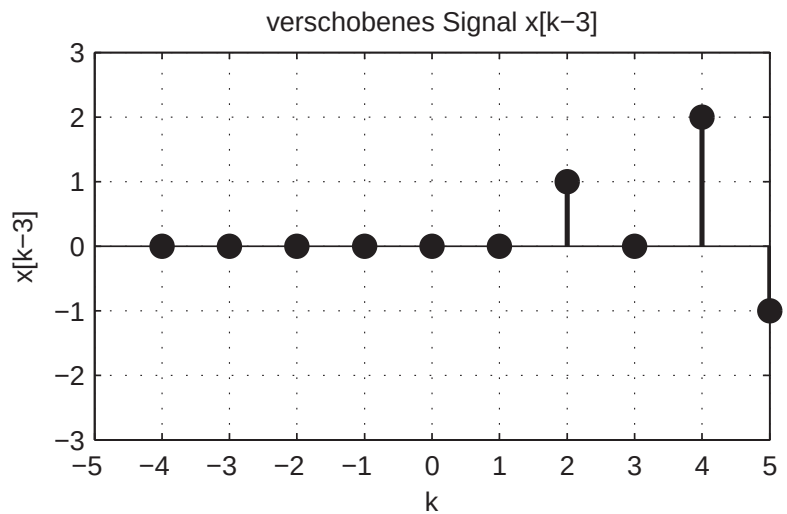

um k=0 gespiegeltes Signal x[−k]
x[−k]
k

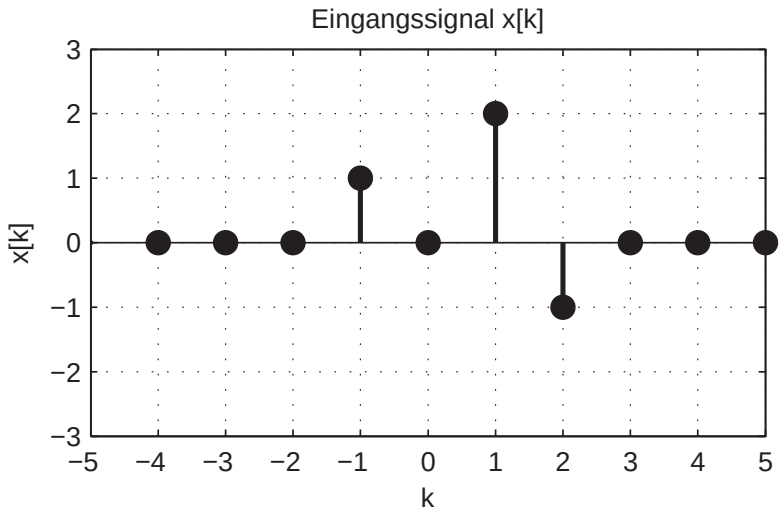

Impulsantwort h[k]
h[k]
k

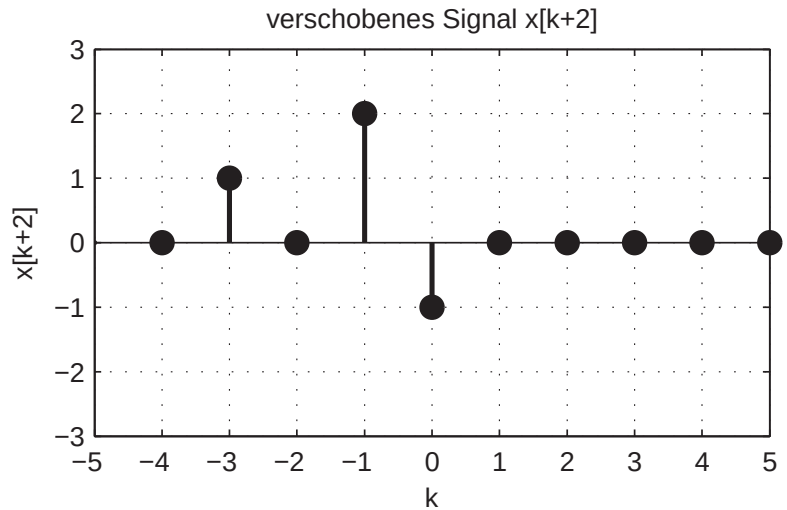

Eingangssignal x[k]
x[k]
k

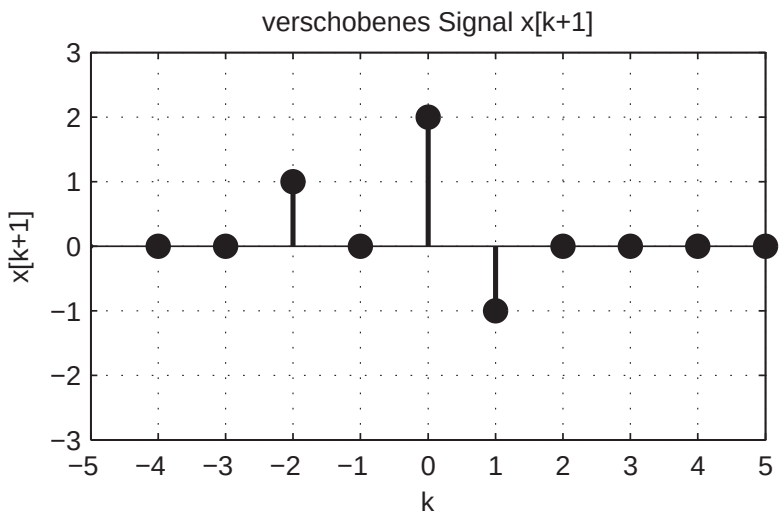

verschobenes Signal x[k−2]
x[k−2]
k

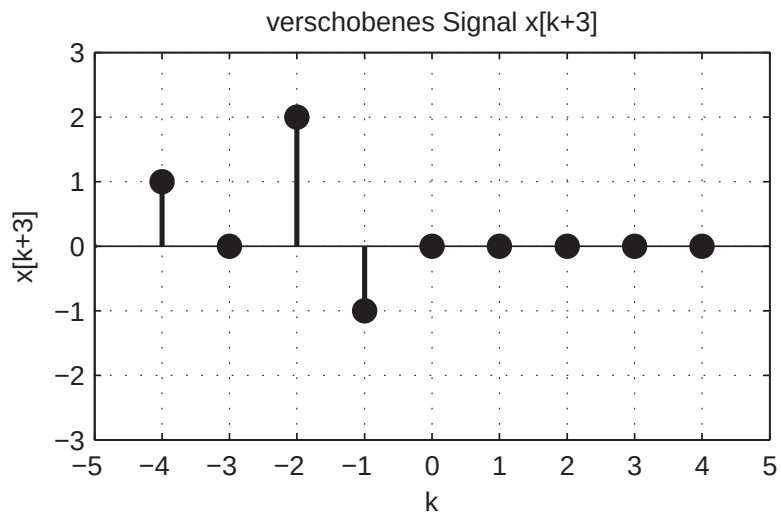

verschobenes Signal x[k−1]
x[k−1]
k

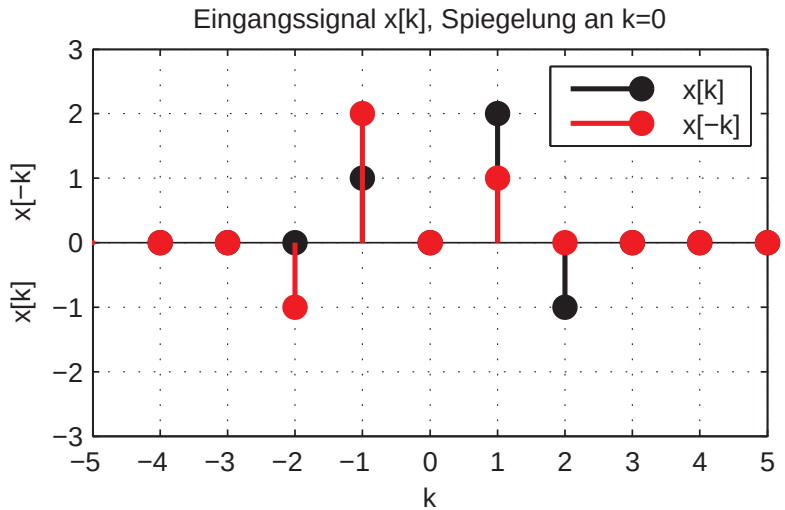

verschobenes Signal x[k−3]
x[k−3]
k

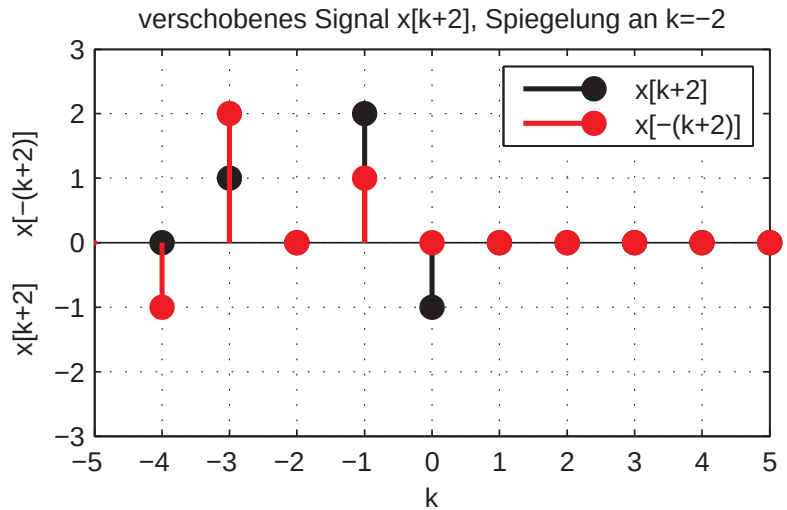

Eingangssignal x[k]
x[k]
k

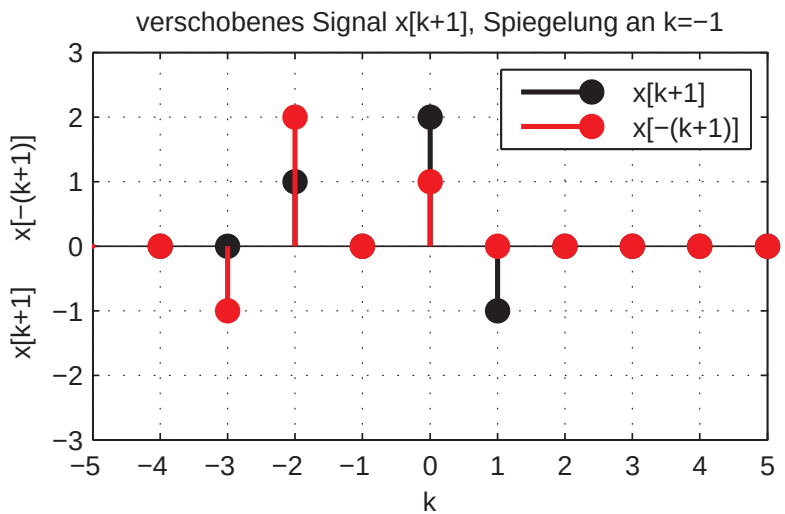

verschobenes Signal x[k+2]
x[k+2]
k

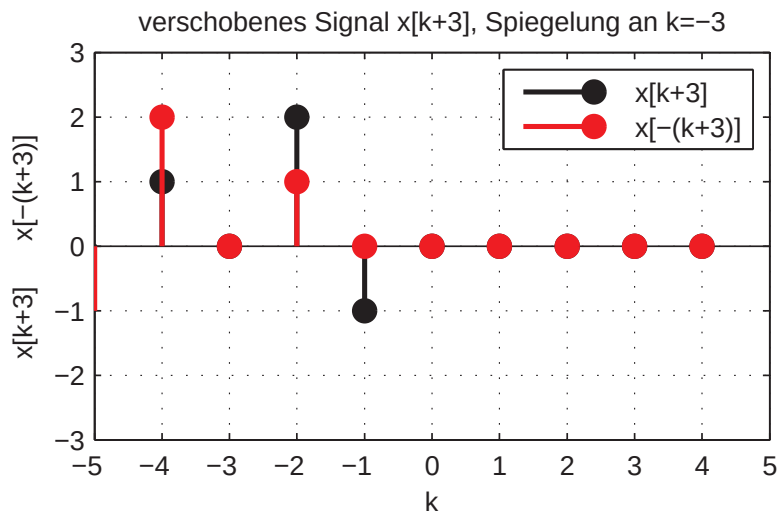

verschobenes Signal x[k+1]
x[k+1]
k

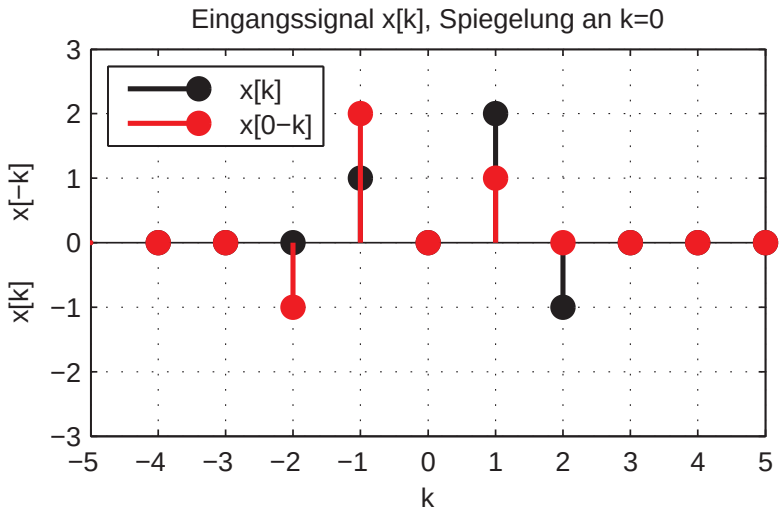

verschobenes Signal x[k+3]
x[k+3]
k

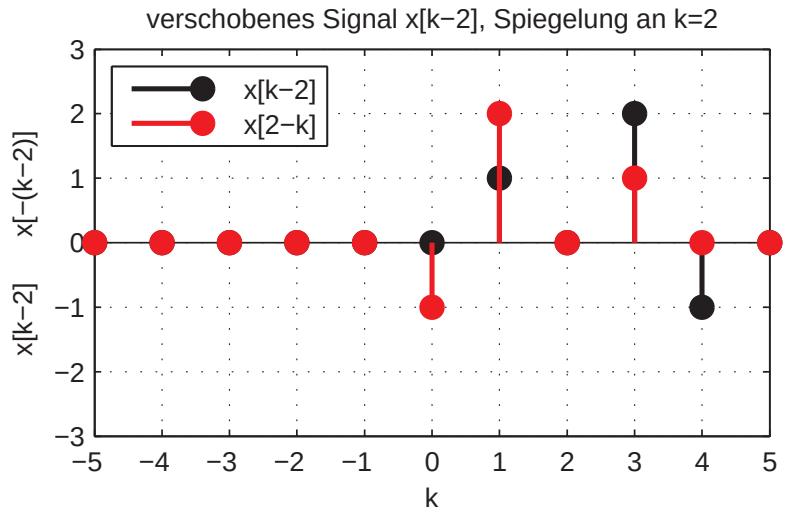

Eingangssignal x[k], Spiegelung an k=0
x[k]
x[−k]
x[−k]
x[k]
k

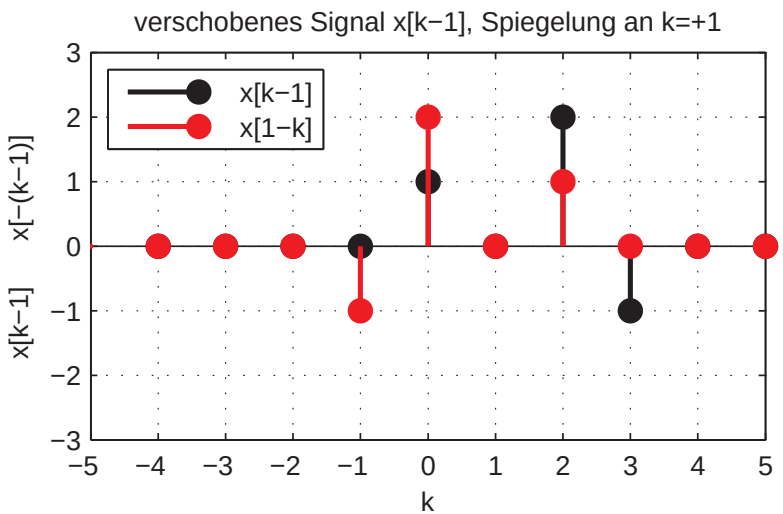

verschobenes Signal x[k+2], Spiegelung an k=−2
x[k+2]
x[−(k+2)]
x[−(k+2)]
x[k+2]
k

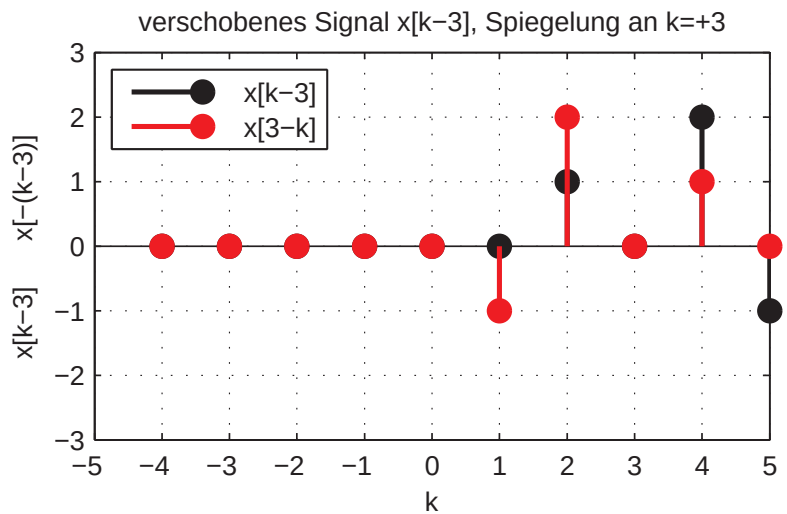

verschobenes Signal x[k+1], Spiegelung an k=−1
x[k+1]
x[−(k+1)]
x[−(k+1)]
x[k+1]
k


verschobenes Signal x[k+3], Spiegelung an k=−3
x[k+3]
x[−(k+3)]
x[−(k+3)]
x[k+3]
k


Eingangssignal x[k], Spiegelung an k=0
x[k]
x[0−k]
x[−k]
x[k]
k


verschobenes Signal x[k−2], Spiegelung an k=2
x[k−2]
x[2−k]
x[−(k−2)]
x[k−2]
k

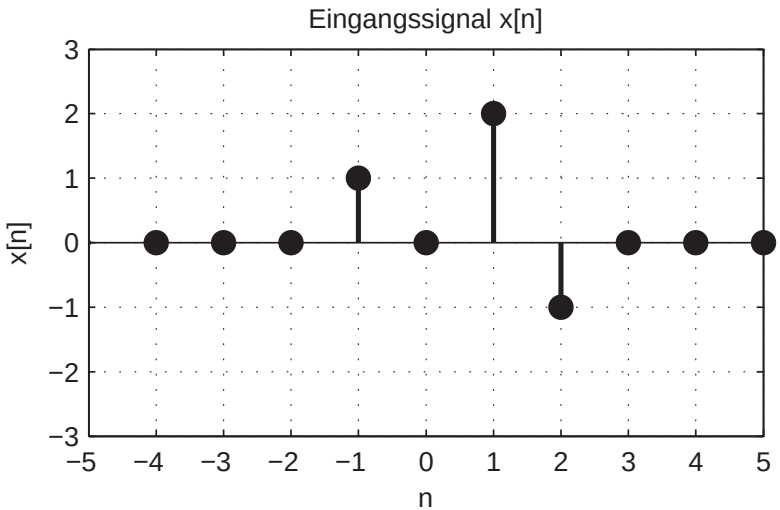

verschobenes Signal x[k−1], Spiegelung an k=+1
x[k−1]
x[1−k]
x[−(k−1)]
x[k−1]
k

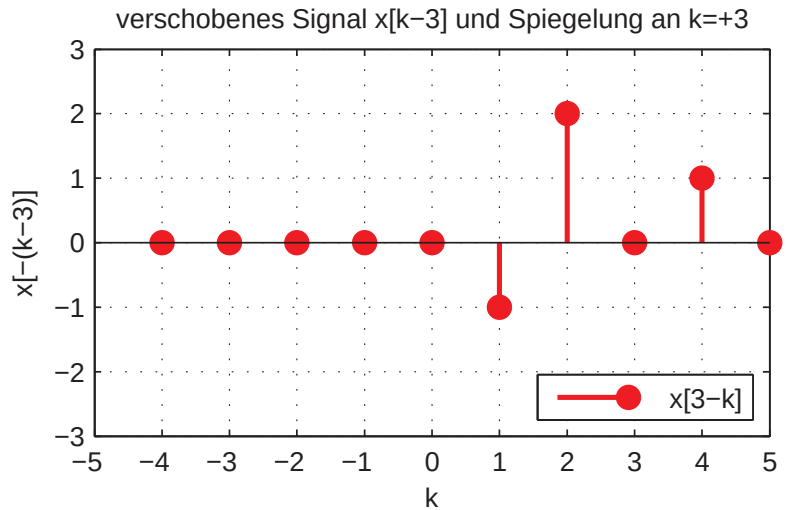

verschobenes Signal x[k−3], Spiegelung an k=+3
x[k−3]
x[3−k]
x[−(k−3)]
x[k−3]
k

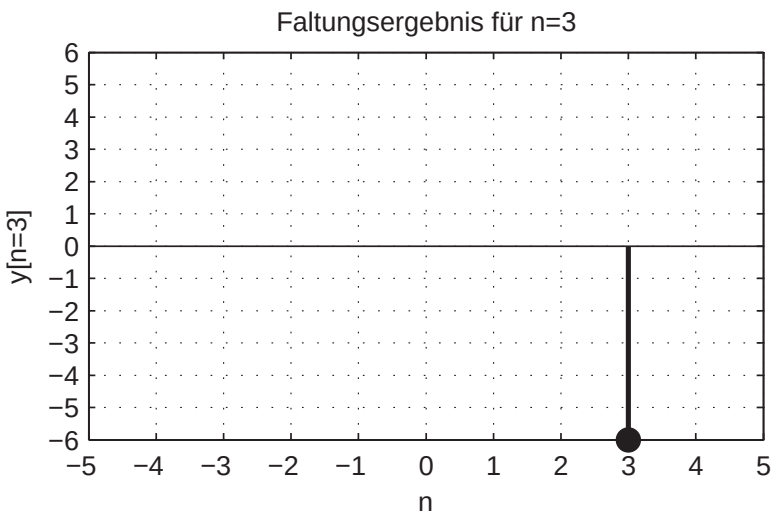
Grafische Veranschaulichung mit

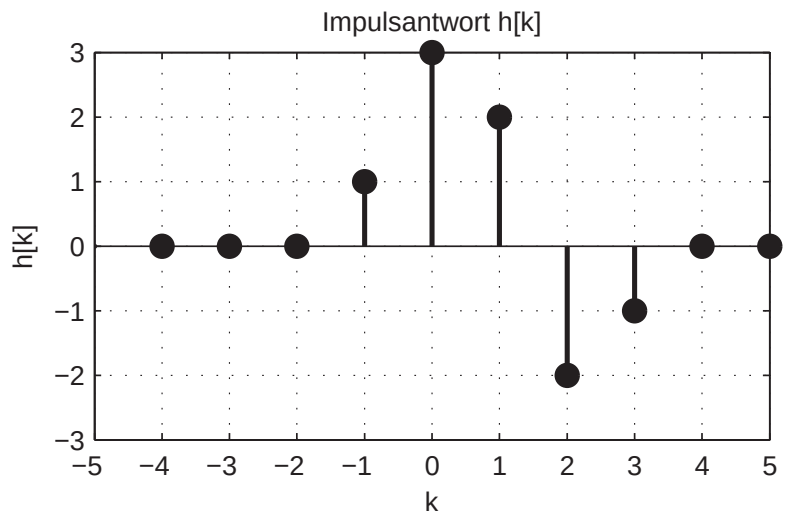
$$y[n] = \sum_{k=-\infty}^{+\infty} h[k] \cdot x[n-k] = (h * x)[n] \tag{13}$$

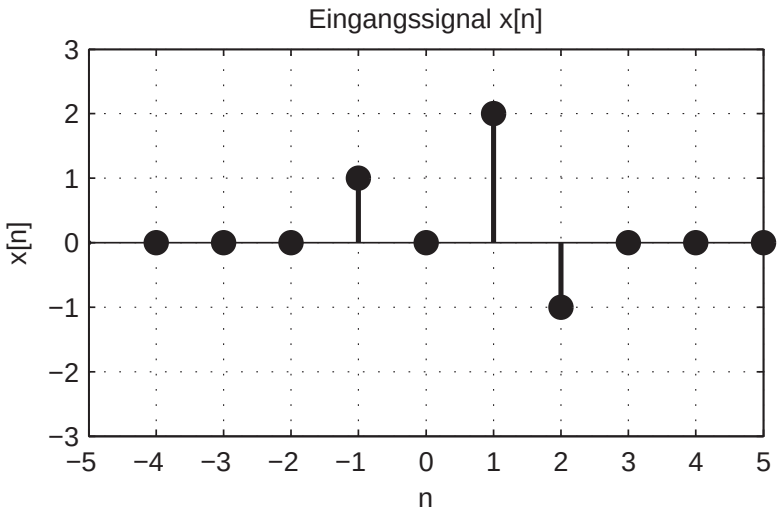
`http://www.nt.e-technik.uni-erlangen.de/~rabe/SYSTOOL/SYSTOOL2.03/conv.htm`

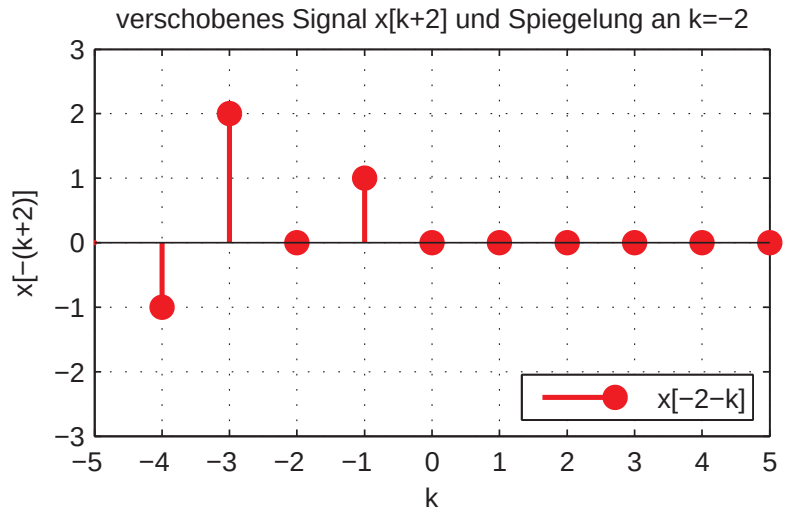

Eingangssignal x[n]
x[n]
n

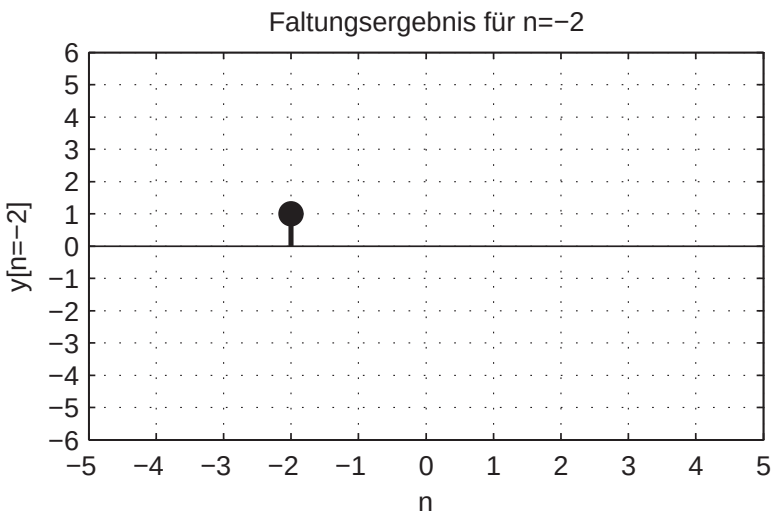

verschobenes Signal x[k−3] und Spiegelung an k=+3
x[−(k−3)]
k
x[3−k]

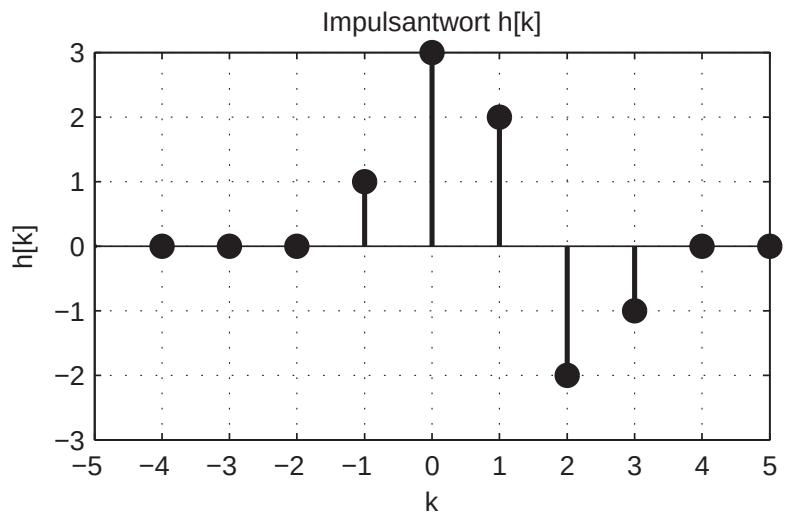

Faltungsergebnis für n=3
y[n=3]
n

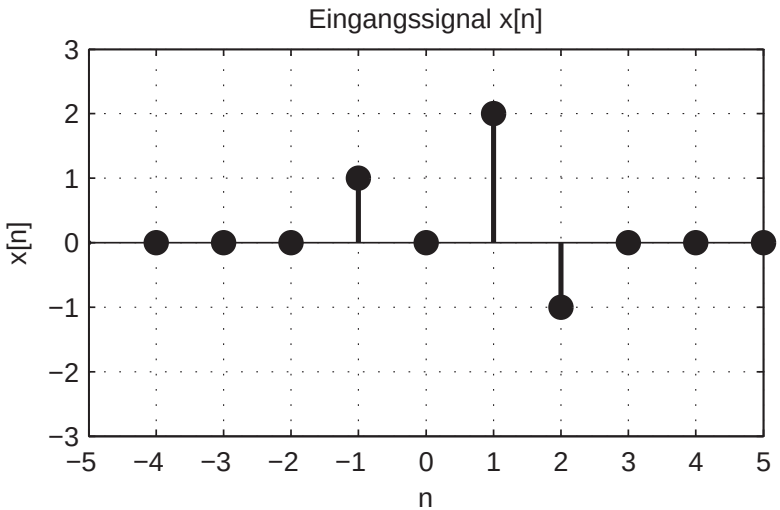

Impulsantwort h[k]
h[k]
k

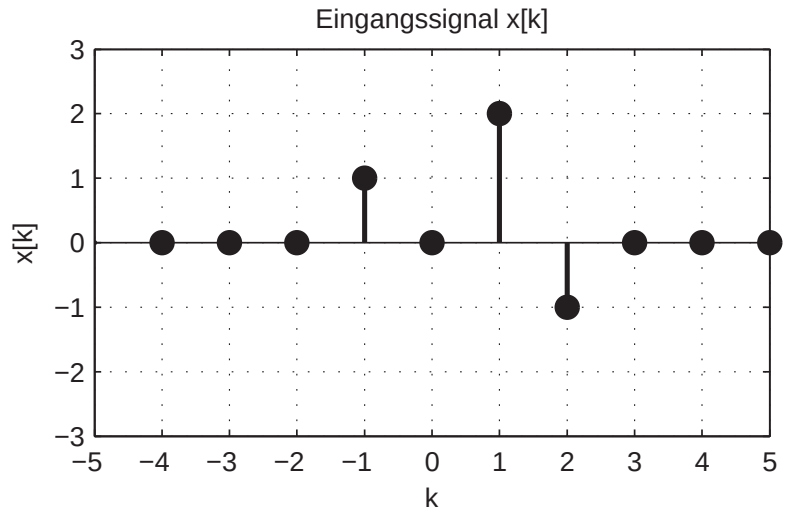

Eingangssignal x[n]
x[n]
n

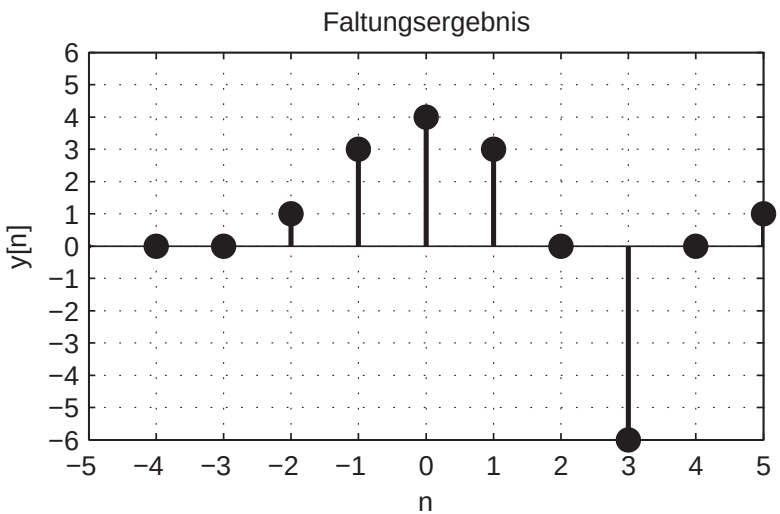

verschobenes Signal x[k+2] und Spiegelung an k=−2
x[−(k+2)]
k
x[−2−k]

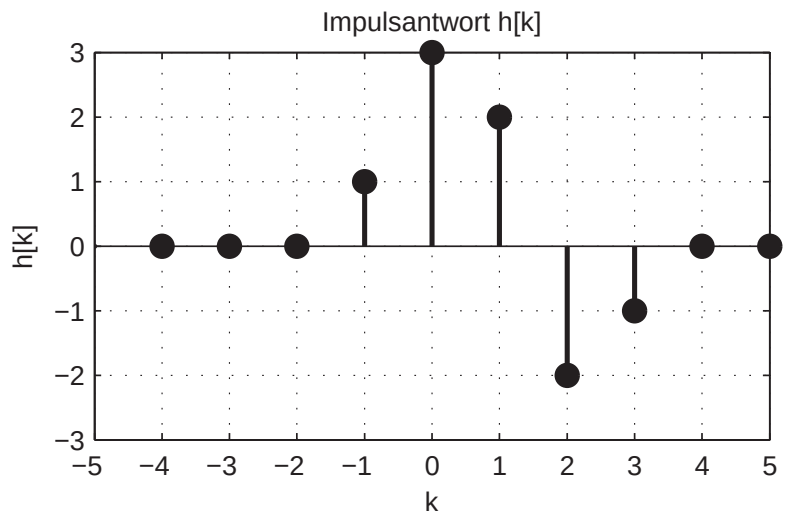

Faltungsergebnis für n=−2
y[n=−2]
n


Impulsantwort h[k]
h[k]
k


Eingangssignal x[n]
x[n]
n


Eingangssignal x[k]
x[k]
k


Faltungsergebnis
y[n]
n


Impulsantwort h[k]
h[k]
k